



TITLE:

Utilization of Data Dependencies and Horizontal Decomposition for the Processing of Cyclic Queries

AUTHOR(S):

Kambayashi, Yahiko; Yoshikawa, Masatoshi

CITATION:

Kambayashi, Yahiko ...[et al]. Utilization of Data Dependencies and Horizontal Decomposition for the Processing of Cyclic Queries. 数理解析研究所講究録 1983, 495: 302-325

ISSUE DATE:

1983-06

URL:

<http://hdl.handle.net/2433/103585>

RIGHT:

Utilization of Data Dependencies and Horizontal Decomposition
for the Processing of Cyclic Queries

Yahiko Kambayashi

上林 彌彦

Masatoshi Yoshikawa

吉川 正俊

Department of Information Science

Faculty of Engineering

Kyoto University

1. Introduction

This paper discusses new query processing procedures utilizing data dependencies which include some of the previously known query processing procedures as special cases.

Since the cost for computing joins is often very high, there are many papers which focus on the processing of joins. With respect to natural joins, queries are classified into two classes, tree and cyclic [BERNG8111]. Tree queries can be processed efficiently [BERNG8111], but cyclic query processing is known to be difficult. It has been shown that any procedure for processing cyclic queries contains a procedure for converting them into tree queries [GOODS8203]. The following procedures realize the conversion.

(1) Relation merging.

(2) Tuple-wise processing.

(3) Attribute addition.

Example of these procedures will be given in Section 3. Method (1) realizes the conversion simply by merging all relations forming a cycle. Method (2) is known as Wong's decomposition approach [WONGY7609], which can be regarded as a conversion process based on tuple-wise horizontal decomposition of a relation. By method (3) the conversion is realized by adding attributes to relations forming a cycle [KAMBY8206]. Although methods (2) and (3) look quite different, in this paper we will show that method (2) can be regarded as a special case of method (3), since horizontal decomposition of relations can be handled by addition of an attribute which identifies each subrelation.

In this paper we will introduce a new procedure for cyclic query processing which utilizes horizontal and vertical decompositions of relations. Vertical decompositions can be realized by functional dependencies (FDs) and multivalued dependencies (MVDs). As join operations are applied to the relations which are processed by selections, the number of tuples is usually less than the original relations and thus there are chances that FDs are satisfied. For a given query, we first list possible dependencies to be utilized and then check these dependencies. Even if such dependencies are not satisfied by a relation we can always obtain a set of relations satisfying the dependencies by horizontal decomposition.

Since our purpose is to perform joins, it is natural to obtain larger relations at each intermediate steps like

procedures (1) and (3). Our procedures, however, show that relation splitting is sometimes very useful to save the processing cost. For such relation splitting only a primitive method (procedure (2)) has been known. Our procedures utilize dependencies which are usually used for database design and give very general relation splitting methods which include procedures (2) and even procedure (3) as special cases.

In Section 2 basic definitions are given. Section 3 shows comparison of the above three procedures and the new procedures introduced in this paper using the same example. Section 4 discusses a query simplification procedure using FDs, and a query processing procedure using FDs and horizontal decomposition. Tuple-wise processing and attribute addition are shown to be special cases of the second procedure. In Section 5 query processing procedures using MVDs (or degenerated MVDs) and horizontal decomposition are shown.

2. Basic Concepts

The following notations of the relational algebra will be used.

Projection: $R[X] = \{t[X] \mid t \in R\}$

θ -selection: $\sigma_{A \theta c} R = \{t \mid t[A] \theta c, t \in R\}$

θ -restriction: $\sigma_{A \theta B} R = \{t \mid t[A] \theta t[B], t \in R\}$

(Here, θ is a comparison operator such as =, >, <, etc.,

and c is a constant value.)

Natural equi-join: $R_1 \bowtie R_j$

$$=\{t | t \in R, t[R_i] \in R_i, t[R_j] \in R_j, R = R_1 \cup R_2\}$$

A query Q , which consists of a qualification q and a target attribute set TA , maps a database state $D(R_1, \dots, R_n)$ into the following relation.

$$(\sigma_q(R_1 \times \dots \times R_n)) [TA] \quad (2-1)$$

A query of which a qualification is a conjunction of clauses of the form $R_i.A_{ik} = R_j.A_{jh}$ is called an equi-join query. If $A_{ik} = A_{jh}$ is satisfied for every clause $R_i.A_{ik} = R_j.A_{jh}$, an equi-join query is called sub-natural. Sub-natural queries of which qualifications contain a clause $R_i.A_{ik} = R_j.A_{jh}$ for every possible combination of A_{ik} and A_{jh} satisfying $A_{ik} = A_{jh}$ are called natural. We will only consider natural equi-join queries in this paper, since any equi-join query can be transformed into a natural one by proper renaming of attributes. For natural equi-join queries, the relation (2-1) becomes equivalent to the following one.

$$(R_1 \bowtie \dots \bowtie R_n) [TA] \quad (2-2)$$

Let $(R_1, \dots, R_n)[R_i]$ be a partial solution of the join for R_i . In this paper, we will develop procedures for joins which will obtain partial solutions for all relations involved in the join. Since target attribute sets are not required to be

considered in our problem, we will use q to represent a query. If the complete result of the join is required, the process to obtain the partial solutions can be used as a preprocessing step.

A query graph $G_q = (V, E, L)$ corresponding to a sub-natural query q is a labeled undirected graph. V is a set of vertices, where v_i in V corresponds to relation R_i referred in q . Two vertices v_i and v_j corresponding to R_i and R_j are connected by an edge iff there is a clause $R_i.A = R_j.A$ in q . The label of the edge is the union of all such A . E is the set of edges and L is the set of labels for E .

Two queries are said to be equivalent if both will produce the same result for any database state. Two query graphs are equivalent if the two corresponding queries are equivalent.

A query is called a tree query if it is equivalent to a query whose query graph is circuit-free; otherwise it is called cyclic [BERNG8111].

Example 1 : Consider the following four relations.

SUPERVISOR(Professor, Student, Approver)

ENROLLMENT(Student, Subject, Approver)

CLASS(Subject, Professor, Approver)

OFFICE(Professor, Room)

Here, we use P , S , J , A and R for attributes Professor, Student, Subject, Approver and Room, respectively. The meaning of these relations is self-evident. We have approvers in the first three

relations. The query graph of the natural equi-join query for this database scheme is shown in Fig.1(a). The edge between CLASS and OFFICE can be removed since there are edges labeled by P between CLASS and SUPERVISOR and between SUPERVISOR and OFFICE. The query is a cyclic query whose cyclic part consists of three relations SUPERVISOR, ENROLLMENT and CLASS. One of the labels A on edges connecting three relations is redundant.

If only three relations except for ENROLLMENT are involved in the query, the query graph becomes the one in Fig.1(b) which represents a tree query.

A semi-join of R_i by R_j is denoted by $R_i \bowtie R_j$ and defined as

$$\begin{aligned} R_i \bowtie R_j &= (R_i \bowtie R_j) [R_i] \\ &= R_i \bowtie R_j [R_i \cap R_j] \end{aligned}$$

For tree queries, there exists an efficient procedure to calculate partial solutions for all relations using semi-joins only [BERNG8111]. If no further application of semi-join changes the contents of a relation, that relation is called irreducible.

A functional dependency (FD): $X \rightarrow Y$ is satisfied in relation R if and only if for any pair of tuples t and t' , $t[X]=t'[X]$ implies $t[Y]=t'[Y]$. Throughout this paper, we assume $X \cap Y = \emptyset$ for an FD: $X \rightarrow Y$. A multivalued dependency (MVD): $X \twoheadrightarrow Y|Z$ is satisfied in relation R (where $XYZ = R$ and $Y \cap Z = \emptyset$) if and only if

$$R = R[XY] \bowtie R[XZ]$$

A degenerated MVD (DMVD) [ARMSD8012] [SAGIF7903] [TANAL7808]:

$X \twoheadrightarrow Y|Z$ is satisfied in relation R (where $X \cap Y = \emptyset$ and $X \cap Z = \emptyset$) if and only if $R = R_1 \cup R_2$ such that the FD: $X \rightarrow Y$ is satisfied in R_1 and the FD: $X \rightarrow Z$ is satisfied in R_2 .

FD and MVD are constraints on R which must be satisfied at all times. A dependency which cannot be derived from the set of such constraints may be satisfied by snapshot data. If a snapshot relation satisfies the condition of an FD (or an MVD), we call it a temporary FD (or a temporary MVD, respectively). Such temporary dependencies are especially important for query processing as will be shown later.

3. Elementary Consideration on Cyclic Query Processing

In this section we will show the basic idea used in our new procedure together with the three basic procedures using one simple example query shown in Fig.1(a). Since an efficient procedure for tree queries is known, we will only show how the query can be transformed into a tree.

(1) Relation merging : This method transforms the given query into a tree by merging all relations in each cyclic part of the query. Since the three relations SUPERVISOR, ENROLLMENT and CLASS form a cycle, we first apply the following join.

SUPERVISOR \bowtie ENROLLMENT \bowtie CLASS

The resulting query graph is shown in Fig.2, which is a tree.

(2) Tuple-wise processing : By this method we first select one relation in a cycle and the query is decomposed into queries each of which processes just one tuple of the selected relation. This is known as the query decomposition method [WONGY7609]. For example, if we select CLASS, the following union of queries is equivalent to the given query.

$$t_i \in \bigcup \text{CLASS} \quad (t_i \bowtie \text{SUPERVISOR} \bowtie \text{ENROLLMENT} \bowtie \text{OFFICE})$$

Each query in the union is a tree query as shown in Fig.3. Here, relation CLASS is regarded as a single tuple relation and it can be decomposed into two relations $t_i[\text{PA}]$ and $t_i[\text{JA}]$. The edge labeled by A connecting the two relations is redundant since there exists another path between the two relations each of whose edge label contains A.

(3) Attribute addition : If we get the query graph shown in Fig.4 by adding attribute labels to edges then the edge between ENROLLMENT and CLASS becomes redundant, that is, the query is converted into a tree. Addition of attributes is realized during the processing of the query. That is, S, A, J values of ENROLLMENT and P, A, J values of CLASS are required to be transmitted to SUPERVISOR. In this particular example the method is equivalent to the relation merging (usually the method is better than the relation merging). The difference is caused by the fact that we can encode J values. By a mutual exchange of J values between ENROLLMENT and CLASS, we can use 1,2,... instead of the real J values in the intersection. The method is very much effective when there are few J values in common in both

relations. A general procedure is given in [KAMBY8206].

(4) Use of dependencies : This method is newly introduced in this paper. If $FD:A \rightarrow J$ is satisfied by CLASS, it can be decomposed into two relations CLASS[A,J] and CLASS[A,P]. The resultant query graph is shown in Fig.5. The query is a tree, since the edge labeled by A between the two projections of CLASS is redundant. By the conventional procedure for tree query processing, we can get all partial solutions for relations in Fig.5. The partial solution for CLASS can be obtained by joining the results for CLASS[A,P] and CLASS[A,J].

Such a decomposition of CLASS is possible when $MVD:A \twoheadrightarrow P|J$ is satisfied. $FD:A \rightarrow P$ and $FD:A \rightarrow J$ are special cases of the MVD. Degenerated $MVD:A \twoheadrightarrow P|J$ is another special case. Note that if $MVD:A \twoheadrightarrow P|J$ which is not FD or DMVD is used, we may not be able to get the partial solution for CLASS by joining the two projections (Details about this problem will be discussed in Sections 5).

4. Query Processing Utilizing Functional Dependencies

4.1 Query Simplification Utilizing FDs

Theorem 1 : If a query graph G contains an edge $e = \langle R_i, R_j \rangle$ with label Z such that

(1) $FD: X \rightarrow Y$ is satisfied by both R_i and R_j , and

(2) $X \subseteq Z$,

then the label of e can be replaced by $Z-Y$ without loss of the equivalence of the query.

Proof : Let G' be the query graph after the replacement of the label of e , also let q and q' be queries corresponding to G and G' , respectively. It is suffice to show the both sides' implication of q and q' .

($q \Rightarrow q'$) Trivial.

($q' \Rightarrow q$) Let $R_i^{q'}$ and $R_j^{q'}$ be partial solutions of R_i and R_j for q' , respectively. To prove the implication, we need to show that $R_i^{q'}[XY] = R_j^{q'}[XY]$ holds. $R_i^{q'}[X] = R_j^{q'}[X]$ holds from the assumption (2). (Remind that our definition of an FD: $X \rightarrow Y$ required the condition $X \cap Y = \emptyset$.) Also $(R_i^{q'}[X] = R_j^{q'}[X]) \Rightarrow (R_i^{q'}[XY] = R_j^{q'}[XY])$ holds from the assumption (1). \square

As a special case, when $X = \emptyset$, Y can be eliminated. Using the above theorem we can simplify a given query graph utilizing FDs. When the FD is satisfied by only one of R_i or R_j (say R_i), the assumption of Theorem 1 is fulfilled by performing a semi-join $R_j \bowtie R_i$. Therefore, when proper FDs hold in relations, there are cases in which the partial solutions of cyclic queries are obtained using only semi-joins.

Besides the FDs given as constraints on the database, we can use various FDs, which is summarized in the following property.

Property 1: The following FDs can be used for query simplification shown in Theorem 1.

(1) FDs in the set of database constraints.

(2) FDs produced by relational operations.

(2-1) If there is a selection operation $\sigma_{A='a'}R$ then there exists an FD: $\emptyset \rightarrow A$, where 'a' is a constant value.

(2-2) If there is a restriction operation $\sigma_{A=B}R$ then there exist FDs: $A \rightarrow B$ and $B \rightarrow A$.

(3) FDs produced by an intersection of the two relations to be joined.

(4) Temporary FDs produced during query processing can be utilized.

In case (3) we need to store the correspondence of X and Y in order to record Y values from the partial solution for a relation whose attribute set contains X.

In the above discussion we considered the situation when FDs are satisfied we will try to use them to simplify the given query. Another approach is first to find a proper set of FDs to simplify the query and then select a proper process such that these FDs are satisfied in the relations. This can be realized by horizontal decomposition of a relation.

Example 3 : We will consider the example used in Section 3. If an FD: $A \rightarrow J$ holds in the relation CLASS, we can eliminate J and the query graph shown in Fig.6 is obtained which is a tree. It is another interpretation of the process discussed in Section 3 (4). We will consider the problem that we know the FD is useful for query simplification and it is not satisfied by CLASS. Fig.7 shows an example of CLASS which does not satisfy the FD. If we decompose the relation into three relations such that one

relation R_1 has the first three tuples and the second relation R_2 has the next two tuples and the third relation R_3 has the last tuple, each of the relations satisfies the FD. We can decompose the query into three subqueries, such that in each subquery CLASS is replaced by R_1 , R_2 or R_3 . The partial solution for the original query is the union of the partial solutions for three subqueries.

We have the following procedure utilizing FDs and horizontal decomposition.

Procedure 1 : (Conversion of a query into a tree utilizing FDs and horizontal decomposition)

(1) For each cyclic part of the query graph apply the following steps.

(2) Select one edge labeled Z .

(3) Let X be the intersection of the labels of edges in the cycle and $Z=XY$ (see Fig.8(a)).

(4) Since if FD: $X \rightarrow Y$ is satisfied the cycle is removed by performing a semi-join (see Fig.8 (b)), we apply horizontal decomposition to one of the two relations corresponding to the terminal nodes of the edge so that each subrelation satisfies the FD.

If the given query has more than one cycle, selection of edges at step (2) can be done by finding all edges not contained in one particular spanning tree. Basic consideration on the selection of such a spanning tree is given in [KAMBY8206].

4.2 Relationship among the Query Conversion Methods

In this subsection, we will compare the query conversion methods presented so far and clarify the relationships among them.

We use the relation in Fig.7 as example again. When the relation is decomposed horizontally so that each of the subrelations satisfies the $FD:A \rightarrow J$, each subquery becomes a tree query and processed separately. However if we handle the subqueries simultaneously, we need to distinguish them. Thus identifiers of subqueries must be attached when semi-joins are performed. For example, when performing a semi-join $SUPERVISOR \bowtie_{PA} CLASS$, a relation with identifiers (IDs) shown in Fig.9 is transmitted to relation SUPERVISOR. Note that the IDs also identify different FDs of the same syntactical form $A \rightarrow J$.

Next we will show that when the left hand side of the FD is \emptyset , this horizontal decomposition approach is essentially the same as the attribute addition method. When the relation CLASS satisfies the $FD:\emptyset \rightarrow J$, the same discussion as in the case of $FD:A \rightarrow J$ holds since the former FD implies the latter. Thus the query can be easily transformed into the tree query shown in Fig.6. If the $FD:\emptyset \rightarrow J$ does not hold on CLASS, we decompose it horizontally again so that each subrelation satisfies the FD. This decomposition is easily achieved by first performing GROUP-BY[J] on CLASS and then regarding each subrelation as a separate relation (see Fig.10 (a), (b)). When we handle these subqueries

simultaneously, we require IDs to distinguish them. However, in this case, values of attribute J can be used as IDs since there is a one-to-one correspondence between subrelations and J-values. Thus J-values are attached to the CLASS[P, A] relation when performing the semi-join SUPERVISOR_{PA} CLASS (see Fig.10 (c), (d); Data compression technique stated in Section 3 (3) is applied in the relation of Fig.10 (d)). This method, however, is the attribute addition approach.

Thus the only difference between horizontal decomposition approach and attribute addition approach when the left hand side of the FD is \emptyset , is that each subquery is considered to be processed separately in the former approach while all of them are regarded to be processed simultaneously in the latter. The horizontal decomposition by FDs with nonempty left hand side is usually more economical than attribute addition, since the number of subrelations usually decreases.

Wong's decomposition approach may be interpreted to mean that we use the horizontal decomposition where each subrelation consists of only one tuple. Since arbitrary FDs are satisfied in these subrelations, we can convert the original query into a tree query if we select at least one relation to be decomposed for each cycle in the query graph.

5. Query Processing Utilizing Multivalued Dependencies

In this section, we will generalize the Procedure 1 in

Section 4 utilizing MVDs.

5.1 Conversion of Queries into Tree Using MVDs

The conversion procedure from a cyclic query to a tree query utilizing FDs (see Fig.8) can be easily generalized to the case utilizing MVDs. Consider the cycle shown in Fig.11 (a).

(1) Select a relation, say R_θ , in the cycle.

(2) Let X be the intersection of labels of edges in the cycle. If an MVD $X' \twoheadrightarrow Y|Z$ holds on R_θ , where $R_\theta = X'YZ$ and $X' \subsetneq X$, then decompose R_θ into two relations $R_\theta^1(X'Y)$ and $R_\theta^2(X'Z)$.

(3) Since the edge between R_θ^1 and R_θ^2 is redundant (see Fig.11 (b)), the original cycle is eliminated. We can use a spanning tree for the selection of relations to be decomposed, when there are many cycles. A general procedure is shown below.

Procedure 2 : (Conversion of a cyclic query into tree utilizing MVDs)

(1) Select a spanning tree in the query graph.

(2) For each edge not contained in the spanning tree, select one relation between relations corresponding to the two nodes incident to the edge.

(3) Determine the MVD which should hold on the relation corresponding to the node chosen in (2), in order to remove the cycle.

(4) Decompose the relation using the MVD.

(5) Repeat (2)-(4) for every edge not contained in the spanning

tree.

5.2 Query Processing Using DMVDs

In this subsection, a new query processing approach using DMVDs is introduced. First an example is given. We will use the query which consists of three relations SUPERVISOR, ENROLLMENT and CLASS in Example 1. Assume a DMVD: $A \Rightarrow P|J$ is satisfied in the relation CLASS. Also let $CLASS_1$ and $CLASS_2$ be horizontally partitioned subrelations in which FDs: $A \rightarrow P$ and $A \rightarrow J$ are satisfied, respectively (Fig.12 (a)). Following the discussions of subsection 5.1, the original query can be converted to tree queries of Fig.12 (b) and (c). Each query graph corresponds to $CLASS_1$ and $CLASS_2$, so the result of the original query is the union of the results of these two queries.

As in the case of FDs, we can generalize the idea for the situation where the original relation does not satisfy a DMVD by horizontally partitioning relations.

5.3 Query Processing Using MVDs

Although MVDs can also be utilized for query conversion, there is a qualitative difference between the usage of FDs (or DMVDs) and that of MVDs. The difference is whether the partial solutions of all relations can be obtained using only semi-joins or not. When MVDs are utilized for the conversion, the partial solution for the relation decomposed by an MVD cannot in general

be obtained by semi-joins only. The following example clarify this.

Example 4 : Consider a query shown in Fig.13 (a). If MVD: $D \twoheadrightarrow B|C$ is satisfied on R_3 , it can be decomposed into two relations $R_3^1(CD)$ and $R_3^2(BD)$. Thus the query can be converted into the tree as shown in Fig.13 (b). Therefore the partial solutions of R_1 , R_2 , R_3^1 and R_3^2 can be easily obtained, while that of R_3 is not. To confirm this, just observe the contents of the relation in Fig.13 (c) where all three relations are irreducible and R_3 satisfies the MVD: $D \twoheadrightarrow B|C$, yet R_3 is not the partial solution. Therefore the method can be used, if the target attributes do not contain both B and C.

In the case of FDs we used horizontal decomposition when FDs are not satisfied. For MVDs we can use overlapped decomposition called cover. If an MVD is not satisfied in the original relation, the covering of the relation is obtained such that each cover satisfies an MVD (see Fig.14).

Acknowledgments

The authors are grateful to Professor Shuzo Yajima and members of Yajima Lab. for helpful discussions.

References

[ARMSD8012] Armstrong, W.W. and Delobel, C., "Decompositions and Functional Dependencies in Relations.", ACM TODS, Vol.5, No.4, pp.404-430, Dec. 1980.

[BERNG8111] Bernstein, P.A. and Goodman, N., "Power of Natural SemiJoins", SIAM J. Comput., Vol.10, No.4, pp.751-771, Nov. 1981.

[GOODS8203] Goodman, N. and Shmueli, O., "The Tree Property is Fundamental for Query Processing (Extended Abstract)." Proc.1st ACM SIGACT-SIGMOD Symposium on PODS, pp.40-48, Mar.1982.

[KAMBY8206] Kambayashi, Y., Yoshikawa, M. and Yajima, S., "Query Processing for Distributed Databases Using Generalized Semi-Joins.", Proc. ACM SIGMOD Intl. Conf. on Management of Data, pp.151-160, June 1982.

[SAGIF7903] Sagiv, Y. and Fagin, R., "An Equivalence between Database Dependencies and a Subclass of Propositional Logic.", IBM Res. Rep., RJ2500, Mar. 1979.

[TANAL7808] Tanaka, K., Le Viet, C., Kambayashi, Y. and Yajima, S., "A File Organization Suitable for Relational Database Operations.", Lecture Notes in Computer Science 75, pp.193-227, Aug. 1978.

[WONGY7609] Wong, E. and Youseffi, K., "Decomposition - A Strategy for Query Processing.", ACM TODS, Vol.1, No.3, pp.223-241, Sep. 1976.

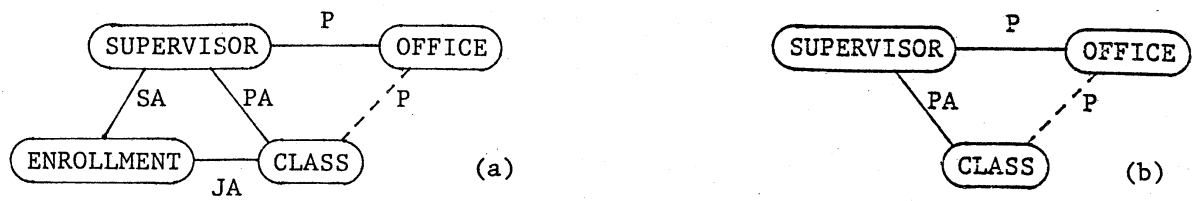


Fig.1 A cyclic query and a tree query

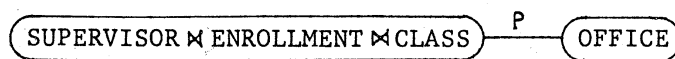


Fig.2 Conversion to a tree query by relation merging

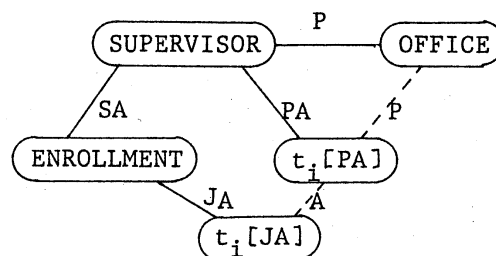


Fig.3 Each query of the tuple-wise processing

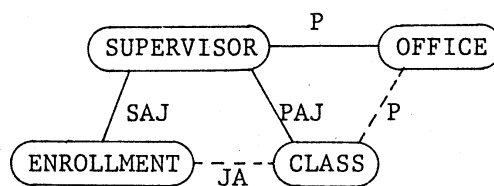


Fig.4 Conversion to a tree query by adding attributes

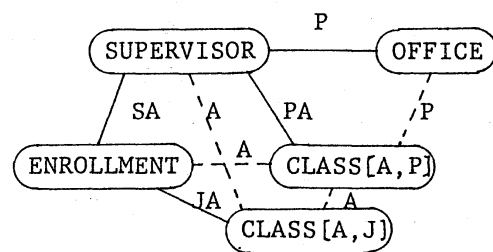
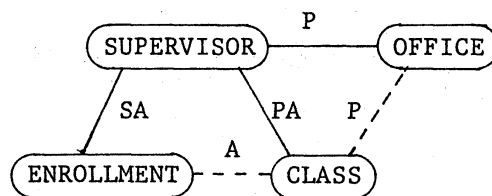


Fig.5 Use of a dependency for query conversion

Fig.6 Use of an FD : $A \rightarrow J$ for query conversion

CLASS

J	P	A
j ₁	p ₂	a ₁
j ₄	p ₁	a ₂
j ₂	p ₂	a ₃
j ₂	p ₁	a ₁
j ₃	p ₁	a ₃
j ₁	p ₂	a ₃

Fig.7 Horizontal decomposition of a relation

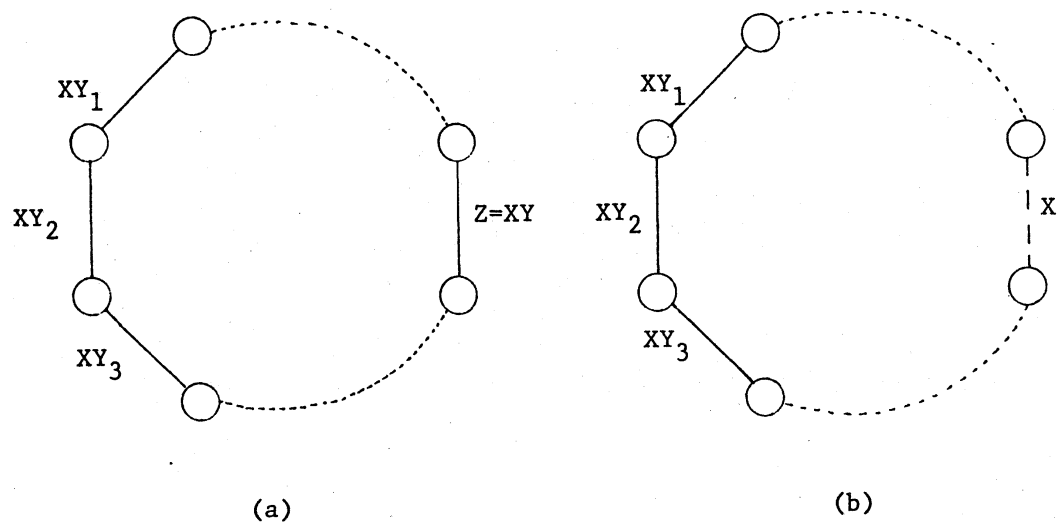


Fig.8 Removal of a cycle

CLASS		
<u>ID</u>	P	A
1	P_2	a_1
	P_1	a_2
	P_2	a_3
2	P_1	a_1
	P_1	a_3
3	P_2	a_3

Fig.9 Attachment of ID's to subrelations

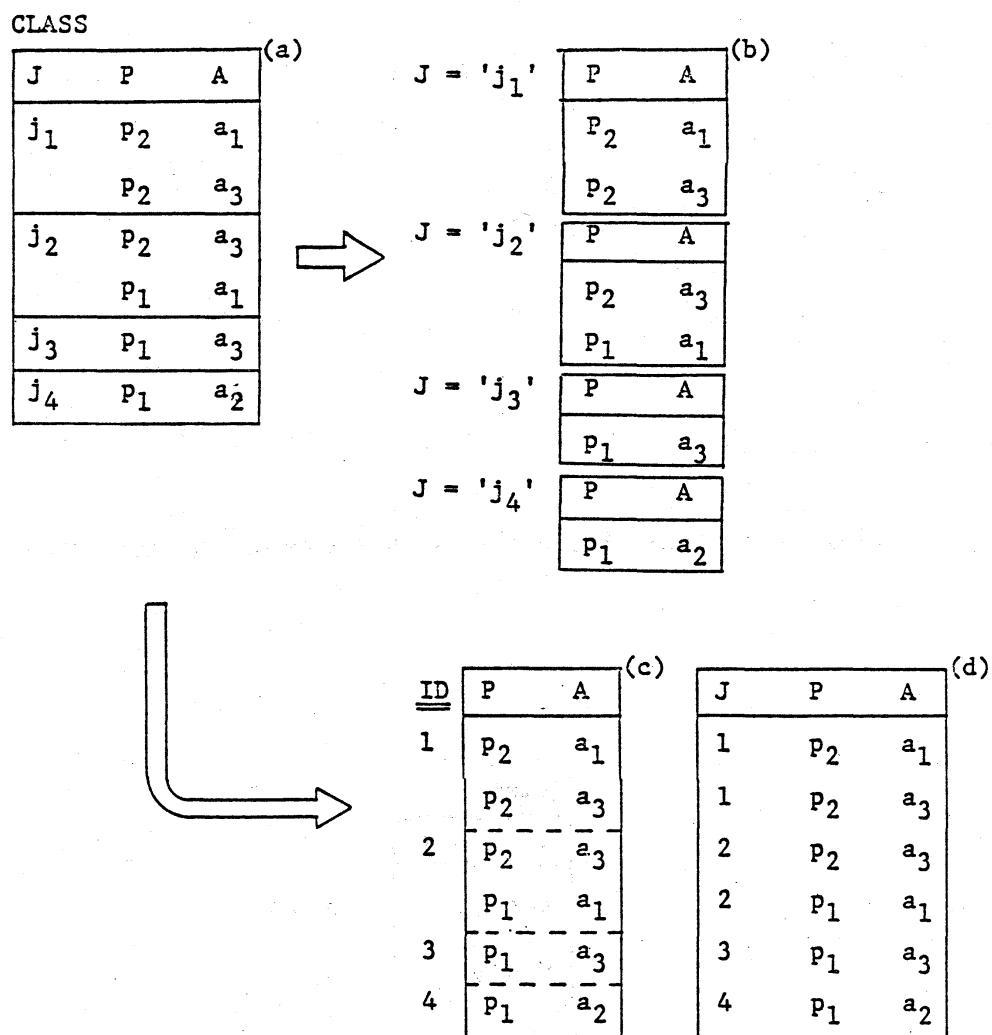


Fig.10 Horizontal decomposition method utilizing $\phi \rightarrow J$ and attribute addition method

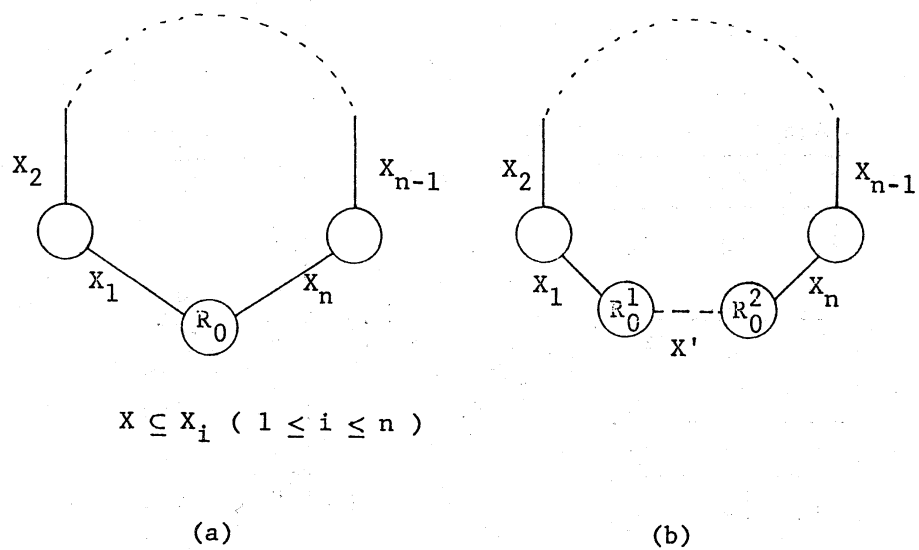
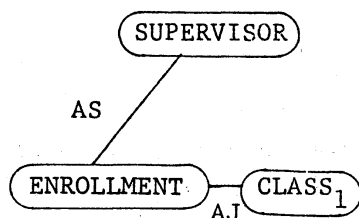


Fig.11 Removal of a cycle by an MVD-based decomposition

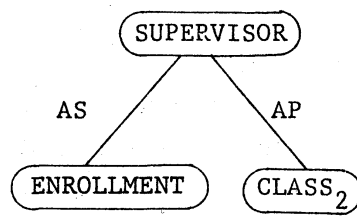
CLASS		
J	P	N
CLASS ₁		
CLASS ₂		

$A \rightarrow P$
 $A \rightarrow J$

(a)

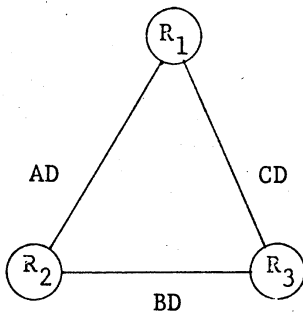


(b)

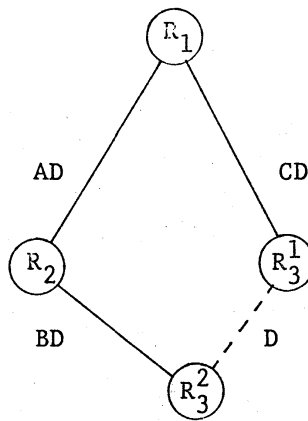


(c)

Fig.12 Query processing using a DMVD



(a)



(b)

R_1			R_2		
A	C	D	A	B	D
a_1	c_1	d_1	a_1	b_1	d_1
a_2	c_2	d_1	a_2	b_2	d_1

(c)

R_3		
B	C	D
b_1	c_1	d_1
b_1	c_2	d_1
b_2	c_1	d_1
b_2	c_2	d_1

Fig.13
The case when not all
the partial solutions
are obtained by semi-
joins only

B	C	D
b_1	c_1	d_1
b_1	c_2	d_1
b_2	c_1	d_1
b_2	c_2	d_1
b_2	c_3	d_1
b_3	c_2	d_1
b_3	c_3	d_1

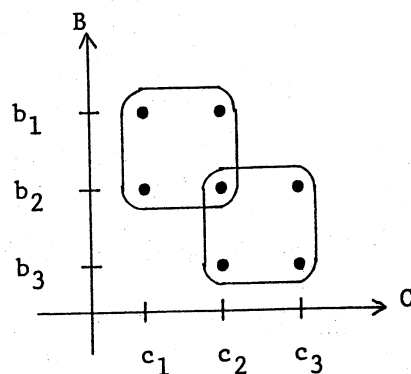


Fig.14 Covering of the relation